We have the following law of motion for unemployment, both in the partial and in the general equilibrium search model.

$$u_{t+1} = u_t + \lambda (1 - u_t) - f_t u_t$$

where $f = H = 1 - F(w_R)$ in the PE model and $f = p(\theta)$ in the GE model.

First, we can use this equation to calculate the steady state unemployment:

$$u_{t+1} = u_t = \bar{u} \Rightarrow \bar{u} = \lambda (1 - \bar{u}) + f\bar{u} \Leftrightarrow \bar{u} = \frac{\lambda}{\lambda + f}$$

Second, we can re-write the law of motion in terms of deviations from steady state, and use it to calculate the half-life of unemployment adjustment.

$$\begin{aligned}
u_{t+1} - \bar{u} &= \lambda + (1 - \lambda - f)(u_t - \bar{u}) + (1 - \lambda - f)\bar{u} - \bar{u} \\
&= (1 - \lambda - f)(u_t - \bar{u}) + \lambda - (\lambda + f)\bar{u} \\
&= (1 - \lambda - f)(u_t - \bar{u}) + \lambda - (\lambda + f)\frac{\lambda}{\lambda + f} \\
&= (1 - \lambda - f)(u_t - \bar{u})
\end{aligned}$$

Iterating backward:

$$\begin{aligned}
u_{t+\tau} - \bar{u} &= (1 - \lambda - f)(u_{t+\tau-1} - \bar{u}) = (1 - \lambda - f)^2 (u_{t+\tau-2} - \bar{u}) \\
&= (1 - \lambda - f)^{\tau} (u_t - \bar{u})
\end{aligned}$$

The half life $t_{1/2}$ is defined as the number of time periods after which the unemployment rate goes half the way back to steady state. So, if $\tau = t_{1/2}$ then $u_{t+\tau} - \bar{u} = \frac{1}{2}(u_t - \bar{u})$. Therefore,

$$u_{t+\tau} - \bar{u} = (1 - \lambda - f)^{t_{1/2}} (u_t - \bar{u}) = \frac{1}{2}(u_t - \bar{u})$$

$$\begin{aligned}
(1 - \lambda - f)^{t_{1/2}} &= \frac{1}{2} \\
t_{1/2} \log(1 - \lambda - f) &= -\log 2 \\
t_{1/2} &= -\frac{\log 2}{\log(1 - \lambda - f)} \simeq -\frac{\log 2}{-\lambda - f} = \frac{\log 2}{\lambda + f}
\end{aligned}$$