Accounting for Mismatch Unemployment

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Why are the latest results different from those in previous versions of the paper?

1 Introduction

In earlier versions of this paper, we found that mismatch was primarily driven by wage frictions, with very little role for worker and job mobility frictions. In the current version, we find a much larger contribution of job mobility frictions, and to some extent worker mobility frictions, and very little role for wage frictions. These differences are due to a change in the expressions for worker and firm surplus. This note aims to clarify this.

Below, we first derive the correct expression for surplus, and then change the assumptions of the model to derive the old expression. We argue why the assumptions needed to derive the old expression are not plausible, and then explain how the difference in the expressions for surplus causes the difference in the results.

For simplicity, we focus on worker surplus. The issue is completely symmetric for firm surplus. We derive the expression for surplus in the context of our model, but appendix A gives the same derivation in the context of a standard search model, e.g. Pissarides (1985), extended with multiple segments, which is very similar to our model and may be more familiar to readers.

2 Derivation of the correct expression for worker surplus

The optimization problem of the household, as described in appendix A.2.1, gives rise to the following Bellman equation

$$V\left(\{n_{it}\}\right) = \max_{\{u_{it}\}} \left\{ w_{it} n_{it} + b_{it} u_{it} + \beta E_t V\left(\{n_{it+1}\}\right) \right\} \tag{1}$$

where

$$n_{it+1} = \left(1 - \delta_i\right) n_{it} + p_{it} u_{it} \tag{2}$$

subject to the constraint that the total number of workers is fixed (with multiplier λ_t^u assigned to the contraint),[1]

$$\sum_i u_{it} = 1 - \sum_i n_{it} \tag{3}$$

[1] In the paper, λ^u is home production of workers not participating in the labor force, constraint (3) is dropped and a term $+\lambda^u \sum_i \left(1 - n_{it} - u_{it}\right)$ enters directly in the objective function. This formulation is mathematically identical, except that in the version with the constraint used, the multiplier is time-varying. For the purposes of this note, we use the version with contraint, which makes it clearer where the difference with the old expression is.

and taking wages w_{it} and job-finding probabilities p_{it} as given.

From the envelope condition,

$$V_i(\{n_{it}\}) = w_{it} - \lambda_t^u + \beta(1-\delta_i) V_i(\{n_{it+1}\})$$

we get worker surplus, i.e. the expected net present value to the household of having one more worker being employed.

$$S_{it}^W = \beta E_t[w_{it+1} - \lambda_t^u] + \beta(1-\delta_i) E_t S_{it+1}^W \tag{4}$$

$$= \beta \sum_{s=0}^{\infty} \beta^s (1-\delta_i)^s E_t\left[w_{it+s+1} - \lambda_{t+s+1}^u\right] = \frac{w_{it} - \lambda_t^u}{r+\delta_i} \tag{5}$$

where the last equality follows if we assume that wages and multipliers follow a random walk.

3 Derivation of the old (incorrect) expression for S_{it}^W

Now suppose that when workers become unemployed, they remain in the labor market segment where they were previously employed. Then, constraint (3) is replace by

$$u_{it} = \ell_{it} - n_{it} \tag{6}$$

where ℓ_{it} is the total labor force of workers (employed and unemployed) in segment i, which is exogenous.

Substituting this constraint into the objective function, the Bellman equation is now given by

$$V(\{n_{it}\}) = \max\{w_{it}n_{it} + b_{it}(\ell_{it} - n_{it}) + \beta V(\{n_{it+1}\})\} \tag{7}$$

where

$$n_{it+1} = (1-\delta_i) n_{it} + p_{it}(\ell_{it} - n_{it}) \tag{8}$$

Worker surplus follows again from the partial derivative of the Bellman equation with respect to n_{it}.

$$V_i(\{n_{it}\}) = w_{it} - b_{it} + \beta(1-\delta_i - p_{it}) V_i(\{n_{it+1}\}) \tag{9}$$

$$S_{it}^W = \beta E_t[w_{it+1} - b_{it+1}] + \beta(1-\delta_i - p_{it}) E_t S_{it+1}^W \tag{10}$$

$$= \beta \sum_{s=0}^{\infty} \beta^s \prod_{\tau=0}^{s} (1-\delta_i - p_{it+\tau}) E_t[w_{it+s+1} - b_{it+s+1}] = \frac{w_{it} - b_{it}}{r+\delta_i + p_{it}} \tag{11}$$

where the last equality now not only assumes that wages and unemployment benefits, but also surplus S_{it}^W follows a random walk.

4 Why the old expression is not correct

Constraint (6) implies that the household can effectively no longer choose the allocation of unemployed workers over labor market segments, but faces an allocation of unemployed workers that depends on the state variables $\{n_{it}\}$ and the exogenous allocation of the labor force.[2] The assumption that a worker who becomes unemployed must remain in the segment where she was previously employed, imposes full segmentation of the labor market. Full segmentation means that worker mobility frictions between segments are prohibitively large, whereas in the no-mismatch equilibrium worker mobility frictions are zero. Therefore, equation (11) is not the correct expression for worker surplus in the no-mismatch equilibrium.[3]

5 Why the difference matters

Comparing expressions (5) and (11) for worker surplus, the differences may not look particularly important. In both cases, worker surplus is the value of an annuity of wages net of an opportunity cost (the shadow value of an additional worker or home production of an unemployed worker). And in both cases the discount rate includes not only the rate of time preference, but also the probability that an employed worker gets separated from her job.

The main difference is that in the old expression (11), the discount rate also includes the probability that an unemployed worker finds a job. The intuition is that the difference in value between an employed and an unemployed worker disappears not only when the employed worker loses her job, but also when the unemployed worker finds one. In expression (5), the option value of an unemployed worker to find a job is not segment-specific and is included in the multiplier λ_t^u.

How does the different denominator in the old expression for surplus affect our results? The importance of worker mobility frictions is measured by the dispersion in the wedge $\hat{\gamma}_{it}^{WM}$ in the worker mobility condition, which is the same in both models:

$$\hat{p}_{it} + \hat{S}_{it}^W = \hat{\gamma}_{it}^{WM} \tag{12}$$

The job-finding probability p_{it} in the denominator of expression (11) for $\hat{S}_{it}^W$ introduces a negative correlation between $\hat{S}_{it}^W$ and $\hat{p}_{it}$. This negative correlation decreases the size

[2] If we allow the household to choose ℓ_{it} subject to the constraint $\sum_i \ell_{it} = 1$, then we are back in the previous case.

[3] As an aside, if we subsitute the WM condition, $p_{it} S_{it}^W = \lambda_t^u - b_{it} \Rightarrow p_{it} = (\lambda_t^u - b_{it})/S_{it}^W$, into expression (11) and rearrange,

$$S_{it}^W = \frac{w_{it} - b_{it}}{r + \delta_i + p_{it}} = \frac{(w_{it} - b_{it}) S_{it}^W}{(r + \delta_i) S_{it}^W + \lambda_t^u - b_{it}} \Rightarrow (r + \delta_i) S_{it}^W + \lambda_t^u - b_{it} = w_{it} - b_{it} \Leftrightarrow S_{it}^W = \frac{w_{it} - \lambda_t^u}{r + \delta_i}$$

then we get back expression (5) for worker surplus. Thus, if the WM condition held true in the data, it would not matter which expression for surplus we used. However, the WM condition does of course not hold true if there are any worker mobility frictions.

of the worker mobility wedges $\hat{\gamma}_{it}^{WM}$. Therefore, we found very little role for worker mobility frictions in earlier version of our paper that used expression (11). The negative correlation between $\hat{S}_{it}^{W}$ and $\hat{p}_{it}$ is much smaller if we use the correct expression (5) for $\hat{S}_{it}^{W}$, which explains why our estimates for the role of worker mobility frictions are much larger in the newer versions of the paper.

6 Conclusion

The result in earlier versions of our paper that there is virtually no role for worker mobility frictions in mismatch unemployment were due to an incorrect expression for worker surplus, and these earlier results are therefore incorrect. However, it is worth mentioning that in the latest versions of the paper, which use the correct expressions for surplus, the role of worker mobility frictions is still much smaller than that of job mobility frictions.

A Worker surplus in a standard search model

To make sure we did not make any mistakes and/or to build a stronger intuition for the difference in the expressions for worker surplus, it may be instructive to rederive expressions (5) and (11) in the context of a standard search model, which is perhaps more familiar to readers. The notation in this appendix follows chapter 1 in Pissarides (2000) insofar it is different from the notation in our model used above.

A.1 Derivation of the old (incorrect) expression

Let W_{it} the value of an employed worker, and U_{it} the value of an unemployed worker in labor market segment i at time t. These values are pinned down by the following Bellman equations.

$$rW_{it} = w_{it} - \delta_i \left(W_{it} - U_{it}\right) \tag{13}$$

$$rU_{it} = b_{it} + p_{it} \left(W_{it} - U_{it}\right) \tag{14}$$

Subtracting the Bellman equation for an unemployed worker U_{it} from that for an employed worker W_{it} and rearranging, we get an expression for worker surplus $S_{it}^{W} = W_{it} - U_{it}$.

$$rS_{it}^{W} = w_{it} - b_{it} - \left(\delta_i + p_{it}\right) S_{it}^{W} \Leftrightarrow S_{it}^{W} = \frac{w_{it} - b_{it}}{r + \delta_i + p_{it}} \tag{15}$$

This is the same expression as (11), which we argued is incorrect because it assumes that the household cannot reallocate unemployed workers across labor market segments.

A.2 Derivation of the correct expression

In this model, the assumption of frictionless worker mobility results in the value of unemployment to be equalized across labor market segments, i.e. $U_{it} = U_t$ for all i. Substituting rU_{it} from (14) into this condition and rearranging, we get the familiar worker mobility condition.

$$p_{it} S_{it}^W = rU_t - b_{it} \tag{16}$$

where $S_{it}^W = W_{it} - U_{it} = W_{it} - U_t$ and rU_t plays the role of λ_t^u in (12).

Subtracting rU_t rather than rU_{it} from the expression for rW_{it} in (13), we get the following expression for worker surplus,

$$rS_{it}^W = w_{it} - rU_t - \delta_i S_{it}^W \Leftrightarrow S_{it}^W = \frac{w_{it} - rU_t}{r + \delta_i} \tag{17}$$

which is the correct expression (5) with $\lambda_t^u = rU_t$ equal to the flow value of unemployment, including the option value of finding a job.

References

Pissarides, C. A. (1985). Short-run equilibrium dynamics of unemployment vacancies, and real wages. *American Economic Review 75*(4), 676–90.

Pissarides, C. A. (2000). *Equilibrium Unemployment Theory* (2nd ed.). Cambridge: MIT Press.